

Coincidence searches of gravitational waves and short gamma-ray bursts

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Abstract Black-hole neutron-star coalescing binaries have been invoked as one of the most suitable scenario to explain the emission of short gamma-ray bursts. Indeed, if the black-hole which forms after the merger, is surrounded by a massive disk, neutrino annihilation processes may produce high-energy and collimated electromagnetic radiation. In this paper, we devise a new procedure, to be used in the search for gravitational waves from black-hole-neutron-star binaries, to assign a probability that a detected gravitational signal is associated to the formation of an accreting disk, massive enough to power gamma-ray bursts. This method is based on two recently proposed semi-analytic fits, one reproducing the mass of the remnant disk surrounding the black hole as a function of some binary parameters, the second relating the neutron star compactness, with its tidal deformability. Our approach can be used in low-latency data analysis to restrict the parameter space searching for gravitational signals associated with short gamma-ray bursts, and to gain information on the dynamics of the coalescing system and on the neutron star equation of state.

1 Introduction

Coalescing binary systems formed by neutron stars (NSs) and/or black holes (BHs) represent one of the most promising sources of gravitational waves (GWs) to be detected by interferometric detectors of second (AdvLIGO/Virgo) and third (the Einstein Telescope, ET) generation [1, 2]. Moreover, these events have recently been

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proposed as a candidate for the central engine of short gamma-ray bursts (SGRB), provided the stellar-mass BH which forms after merging is surrounded by a hot and sufficiently massive accreting disk (see for instance [3] and references therein). Since the electromagnetic emission is produced at large distance from the central engine, it does not give strong information on the source. In addition, the emission is beamed, and consequently these events may not be detected if one is looking in the wrong direction. Conversely, the gravitational wave (GW) emission is not beamed, and exhibits a characteristic waveform (the chirp), which should allow a non-ambiguous identification of the source. GRBs are characterized by a prompt emission, which lasts a few seconds, and an afterglow, whose duration ranges from hours to days. Thus, gravitational wave detection may be used to trigger the afterglow search of GRBs that have not been detected by the on-axis prompt observation and to validate the “jet model” of SGRB. Or, alternatively, the observation of a SGRB may be used as a trigger to search for a coincident GW signal. Indeed, this kind of search has already been done in the data of LIGO and Virgo [4, 5].

However, not all coalescences of compact bodies produce a black hole with an accreting disk sufficiently massive to power a SGRB: it is therefore crucial to devise a strategy to extract those having the largest probability to produce a SGRB. This is one of the purposes of this work. The LIGO-Virgo Collaboration has recently developed a plausible observing schedule, according to which within this decade the advanced detectors, operating under appropriate conditions, will be able to determine the sky location of a source within 5 and 20 deg² [6]. Given the cost of spanning this quite large region of sky to search for a coincident SGRB with electromagnetic detectors, indications on whether a detected signal is likely to be associated with a SGRB are valuable information.

The procedure we propose has several applications. It can be used in the data analysis of future detectors (i) to gain information on the range of parameters that are more useful to span in the low-latency search for GWs emitted by BH-NS sources [7], (ii) for an externally triggered search for GW coalescence signals following GRB observations [4, 5], and (iii) when the binary parameters are measured with sufficient accuracy and in a sufficiently short time to allow for an electromagnetic follow-up to search for off-axis GRB afterglows.

2 Selecting candidates for gamma-ray bursts emission

In the last years a large number of numerical studies of BH-NS coalescence, have allowed to derive two interesting fits. The first [8] gives the mass of the accretion disk, M_{rem} , as a function of the the NS compactness $C = M_{\text{NS}}/R_{\text{NS}}$, where M_{NS} and R_{NS} are the NS gravitational mass and its radius, the dimensionless BH spin, $\chi_{\text{BH}} \in [-1, 1]$, and the mass ratio $q = M_{\text{BH}}/M_{\text{NS}}$:

$$\frac{M_{\text{rem}}}{M_{\text{NS}}^b} = K_1(3q)^{1/3}(1 - 2C) - K_2q C R_{\text{ISCO}} . \quad (1)$$

Here M_{NS}^{b} is the NS baryonic mass which, following [9], we assume to be 10% larger than the NS gravitational mass; R_{ISCO} is the radius of the innermost, stable circular orbit for a Kerr black hole [10]. The two coefficients $K_1 = 0.288 \pm 0.011$ and $K_2 = 0.1248 \pm 0.007$ have been derived [8] through a least-square fit of the results of fully relativistic numerical simulations [11, 12, 13, 14].

M_{rem} is a key parameter in our study. Indeed, neutrino-antineutrino annihilation processes extract energy from the disk [15], and several studies have shown that this process could supply the energy required to ignite a short gamma-ray burst, if $M_{\text{rem}} \in (0.01 \div 0.05)M_{\text{NS}}$ [16]. In the following we shall assume as a threshold for SGRB formation $M_{\text{rem}} = 0.01 M_{\text{NS}}$.

The second fit [17] is a universal relation between the NS compactness C and the tidal deformability $\lambda_2 = -Q_{ij}/C_{ij}$, where Q_{ij} is the NS star traceless quadrupole tensor, and C_{ij} is the tidal tensor,

$$C = 0.371 - 3.9 \times 10^{-2} \ln \bar{\lambda} + 1.056 \times 10^{-3} (\ln \bar{\lambda})^2, \quad \bar{\lambda} = \lambda_2 / M_{\text{NS}}^5. \quad (2)$$

Hereafter, we shall denote by C_λ the NS compactness obtained from this fit.

Let us now assume that the gravitational wave signal emitted in a BH-NS coalescence is detected; a suitable data analysis allows us to find the values of the mass-ratio $q = M_{\text{BH}}/M_{\text{NS}}$, of the chirp mass $\mathcal{M} = (M_{\text{NS}}M_{\text{BH}})^{3/5}/(M_{\text{NS}} + M_{\text{BH}})^{1/5}$, and of the black hole spin χ_{BH} , with the corresponding errors. Knowing $q \pm \sigma_q$ and $\chi_{\text{BH}} \pm \sigma_{\chi_{\text{BH}}}$, using the fit (1) we can trace the plot of Fig. 1 in the $q - C$ plane, for an assigned disk mass threshold, say $M_{\text{rem}} = 0.01M_{\text{NS}}$. This plot allows us to identify the parameter region where a SGRB may occur, i.e., the region $M_{\text{rem}} \gtrsim 0.01M_{\text{NS}}$ (below the fit curve in the figure), and the forbidden region above the fit ($M_{\text{rem}} \lesssim 0.01M_{\text{NS}}$). In addition, we identify four points χ_1, \dots, χ_4 , which are the intersection between the contour lines for $\chi_{\text{BH}} \pm \sigma_{\chi_{\text{BH}}}$ and the horizontal lines $q \pm \sigma_q$. Let us indicate as C_1, \dots, C_4 the corresponding values of the neutron star compactness. Since the fit (1) is monotonically decreasing, $C_1 < C_2 < C_3 < C_4$. At this stage we still cannot say whether the detected binary falls in the region allowed for the formation of a SGRB or not. In order to get this information, we need to evaluate C . As discussed in [18, 19, 20, 21, 22, 23], Advanced LIGO/Virgo are expected to measure the gravitational wave phase with an accuracy sufficient to estimate the NS tidal deformability λ_2 . Thus, using the fit (2), the neutron star compactness C_λ and the corresponding uncertainty σ_{C_λ} can be derived (see [17] for details on how to compute the compactness error).

Knowing the parameters and their uncertainties, the probability that a SGRB is associated to the detected coalescence can now be evaluated. We assume that $(q, C_\lambda, \chi_{\text{BH}})$ are described by a multivariate Gaussian distribution,

$$\mathcal{P}(q, C_\lambda, \chi_{\text{BH}}) = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} \Delta^T \Sigma^{-1} \Delta \right], \quad (3)$$

where $\Delta = (\mathbf{x} - \mu)$, $\mu = (q, C_\lambda, \chi_{\text{BH}})$, and Σ is the covariance matrix. Then, we define the maximum and minimum probability that the binary coalescence produces an accretion disk with mass over the threshold, \bar{M}_{rem} , as

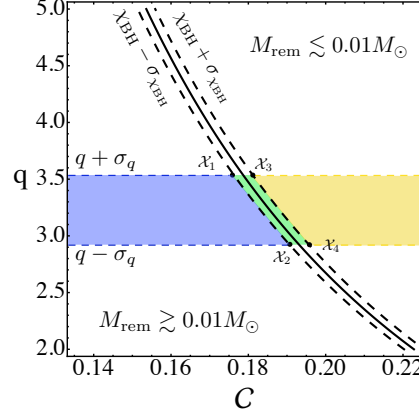


Fig. 1 Contour plot of the fit (1) in the q - C plane, for $M_{\text{NS}} = 1.2M_{\odot}$, $\chi_{\text{BH}} = 0.5$ and $M_{\text{rem}} = 0.01M_{\odot}$. The fit separates the region allowed for SGRB ignition (below the fit curve) from the forbidden region (above the fit). Given the measured values of $q \pm \sigma_q$ and $\chi_{\text{BH}} \pm \sigma_{\chi_{\text{BH}}}$, a detected signal can correspond to a NS with compactness C which falls in one of the regions bounded by the dashed curves. Since C also comes with an error σ_C , in order to infer if it can be associated with a SGRB, we need to evaluate the probability $P(C \leq C_4)$ and $P(C \leq C_1)$ (see text).

$$P_{\text{MAX}}(M_{\text{rem}} \gtrsim \bar{M}_{\text{rem}}) \equiv P(C_{\lambda} \leq C_4) , \quad P_{\text{MIN}}(M_{\text{rem}} \gtrsim \bar{M}_{\text{rem}}) \equiv P(C_{\lambda} \leq C_1) , \quad (4)$$

where $P(C_{\lambda} \leq C_i)$ is the cumulative distribution of Eq. (3), which gives the probability that the measured compactness C_{λ} , estimated through the fit (2), is smaller than an assigned value C_i .

As an illustrative example, we now evaluate the probability that a given BH-NS coalescing binary produces a SGRB, assuming a set of equations of state for the NS matter and evaluating the uncertainties on the relevant parameters using a Fisher matrix approach.

3 The uncertainties on the binary parameters

The accuracy with which future interferometers will measure a set of binary parameters θ is estimated by comparing the gravity-wave data stream with a set of theoretical templates. For strong enough signals, θ are expected to have a Gaussian distribution centered around the true values, with covariance matrix $\text{Cov}^{ab} = (\Gamma^{-1})^{ab}$, where Γ^{ab} is the Fisher information matrix which contains the partial derivatives of the template with respect to the binary parameters [24].

To model the waveform we use the TaylorF2 approximant in the frequency domain, assuming the stationary phase approximation $h(f) = \mathcal{A}(f)e^{i\psi(f)}$ [25]. The post-Newtonian expansion of the phase includes spin-orbit and tidal corrections. It can be written as $\psi(f) = \psi_{\text{PP}} + \psi_{\text{T}}$, i.e. a sum of a point-particle term (see [26, 27])

for the complete expression) and a tidal contribution [28, 19]. The latter is given by

$$\psi_T = -\frac{117\Lambda}{8} \frac{(1+q)^2}{qm^5} x^{5/2} \left[1 + \frac{3115}{1248} x - \pi x^{3/2} + \left(\frac{23073805}{3302208} + \frac{120}{81} \right) x^2 - \frac{4283}{1092} \pi x^{5/2} \right], \quad (5)$$

where $x = (m\pi f)^{5/3}$, $m = M_{\text{BH}} + M_{\text{NS}}$ is the total mass of the system, and Λ is the averaged tidal deformability, which for BH-NS binaries reads [29]: $\Lambda = \lambda_2 \frac{(1+12q)}{26}$.

We consider non-rotating NSs, as this is believed to be a reliable approximation of real astrophysical systems [30, 31]. Therefore, our template is fully specified by 6 parameters¹ $\theta = (t_c, \phi_c, \ln \mathcal{M}, q, \Lambda, \beta)$ where t_c, ϕ_c are the time and phase at the coalescence and β is the 2 PN spin-orbit contribution in ψ_{pp} . We choose the BH spin aligned with the orbital angular momentum. Moreover, since $\chi_{\text{BH}} \leq |1|$, $\beta \lesssim 9.4$; therefore we consider the following prior probability distribution on β : $p^{(0)}(\beta) \propto \exp\left[-\frac{1}{2}(\beta/9.4)^2\right]$.

In our analysis we consider both second and third generation detectors. For AdvLIGO/Virgo we use the ZERO_DET_high_P noise spectral density of AdvLIGO [32], in the frequency ranges [20 Hz, f_{ISCO}]; for the Einstein Telescope we use the analytic fit of the sensitivity curve provided in [33], in the range [10 Hz, f_{ISCO}]. f_{ISCO} is the frequency of the Kerr ISCO including corrections due to NS self-force [34].

We model the NS structure by means of piecewise polytropes, [23]. Indeed we consider four EoS, labeled as 2H,H, HB and B, which denote very stiff, stiff, moderately stiff and soft nuclear matter, respectively. The stellar parameters for $M_{\text{NS}} = (1.2, 1.35)M_{\odot}$, are shown in Table 1.

EoS	$M_{\text{NS}}(M_{\odot})$	C	$\lambda_2 \text{ (km}^5\text{)}$	$M_{\text{NS}}(M_{\odot})$	C	$\lambda_2 \text{ (km}^5\text{)}$
2H	1.2	0.117	75991	1.35	0.131	72536
H	1.2	0.145	21232	1.35	0.163	18964
HB	1.2	0.153	15090	1.35	0.172	13161
B	1.2	0.162	10627	1.35	0.182	8974

Table 1 For each EoS we show the NS mass, the compactness $C = M_{\text{NS}}/R_{\text{NS}}$, and the tidal deformability λ_2 .

4 Numerical results

Following the strategy previously outlined, we compute the minimum and maximum probabilities (4) that the coalescence of a BH-NS system produces a remnant disk with mass above a threshold \bar{M}_{rem} , for the NS models listed in Table 1 and different values of the mass ratio q . The results are given in Table 2-3, for $q = 3$

¹ The signal amplitude $\ln \mathcal{A}$ is uncorrelated with the other variables, so we perform derivatives only with respect to the remaining parameters.

and $q = 7$, black hole spin $\chi_{\text{BH}} = (0.2, 0.5, 0.9)$, $M_{\text{NS}} = (1.2, 1.35) M_{\odot}$, and disk mass thresholds $\bar{M}_{\text{rem}} = 0.01 M_{\text{NS}}$.

For AdvLIGO/Virgo we put the source at a distance of 100 Mpc. For ET the binary is at 1 Gpc. In this case the signal must be suitably redshifted [35, 22], and we have assumed that z is known with a fiducial error of the order of 10% [36].

AdV		$q = 3$			$q = 7$		
$M_{\text{NS}} = 1.2 M_{\odot}$		χ_{BH}			χ_{BH}		
EOS	C_{λ}	0.2	0.5	0.9	0.2	0.5	0.9
2H	0.118	1	1	1	0.4 [0.8-0.9]	1	
H	0.147	[0.6-0.9]	1	1	0.4	0.4	[0.8-0.9]
HB	0.155	[0.5-0.7]	[0.9-1]	1	0.4	0.4	[0.7-0.8]
B	0.164	[0.4-0.6]	[0.7-0.8]	1	0.4	0.4	[0.6-0.7]
$M_{\text{NS}} = 1.35 M_{\odot}$							
2H	0.132	1	1	1	0.3 [0.4-0.5]	1	
H	0.164	[0.4-0.6]	[0.8-0.9]	1	0.4	0.4	0.7
HB	0.173	[0.4-0.5]	[0.6-0.8]	1	0.4	0.4	0.6
B	0.184	[0.4-0.5]	[0.5-0.6]	1	0.4	0.4	0.5

Table 2 We show the probability range $[P_{\text{MIN}}, P_{\text{MAX}}]$ that the coalescence of a BH-NS binary produces a disk mass larger than $\bar{M}_{\text{rem}} = 0.01 M_{\text{NS}}$ for AdLIGO/Virgo (AdV), for binaries with $q = 3$ and $q = 7$, NS masses $(1.2, 1.35) M_{\odot}$, and BH spin $\chi_{\text{BH}} = (0.2, 0.5, 0.9)$. Sources are assumed to be at $d = 100$ Mpc. The star compactness C_{λ} is estimated throughout the universal relation (2).

ET		$q = 3$			$q = 7$		
$M_{\text{NS}} = 1.2 M_{\odot}$		χ_{BH}			χ_{BH}		
EOS	C_{λ}	0.2	0.5	0.9	0.2	0.5	0.9
2H	0.118	1	1	1	[0.3-0.4]	[0.7-0.8]	1
H	0.147	[0.9-1]	1	1	0.3	[0.3-0.4]	1
HB	0.155	[0.7-0.8]	1	1	0.3	0.3	0.9
B	0.164	[0.5-0.6]	1	1	0.3	0.3	[0.7, 0.8]
$M_{\text{NS}} = 1.35 M_{\odot}$							
2H	0.132	1	1	1	0.2	[0.4, 0.5]	1
H	0.164	[0.4-0.6]	1	1	0.3	0.3	0.8
HB	0.173	[0.3-0.4]	0.8	1	[0.3-0.4]	[0.3-0.4]	0.6
B	0.184	[0.3-0.4]	0.6	1	0.4	0.4	[0.5-0.6]

Table 3 Same of Table 2, but for the Einstein Telescope (ET). In this case we assume prototype BH-NS binaries at $d = 1$ Gpc.

The first clear result is that as the BH spin approaches the highest value we consider, $\chi_{\text{BH}} = 0.9$, and for low mass ratio $q = 3$, the probability that a BH-NS coalescence produces a disk with mass above the threshold is insensitive to the NS internal composition, and it approaches unity for all considered configurations. These would be good candidates for GRB production. For the highest mass ratio we consider, $q = 7$, the probability to form a sufficiently massive disk depends on the NS mass and EoS, and on the detector. In particular, it decreases as the EoS softens, and as the NS mass increases. This is a general trend, observed also for smaller values of χ_{BH} . However, when $\chi_{\text{BH}} = 0.9$ the probability that the coalescence is associated to a SGRB is always $\gtrsim 50\%$.

Let us now consider the results for $\chi_{\text{BH}} = 0.2$. If the NS mass is $1.2 M_{\odot}$ the probability that a detected GW signal from a BH-NS coalescence is associated to the formation of a black hole with a disk of mass above threshold is $\gtrsim 50\%$ for both AdvLIGO/Virgo and ET, provided $q = 3$. For larger NS mass, this remains true only if the NS equation of state is stiff (2H or H). High values of q are disfavored.

When the black hole spin has an intermediate value, say $\chi_{\text{BH}} = 0.5$, Table 2 shows that, the NS compactness plays a key role in the identification of good candidates for GRB production, for both detectors. Again large values of the mass ratio yield small probabilities.

The range of compactness shown in Table 2-3 includes neutron stars with radius ranging within $\sim [10 - 15]$ km. From the table it is also clear that if we choose a compactness smaller than the minimum value, the probability of generating a SGBR increases, and the inverse is true if we consider compactness larger than our maximum.

5 Conclusions

The method developed in this paper can be used in several different ways. In the future, gravitational wave detectors are expected to reach a sensitivity sufficient to extract the parameters on which our analysis is based, i.e., chirp mass, mass ratio, source distance, spin and tidal deformability. We can also expect that the steady improvement of the efficiency of computational facilities experienced in recent years will continue, reducing the time needed to obtain these parameters from a detected signal. Moreover, the higher sensitivity will allow us to detect sources in a much larger volume space, thus increasing the detection rates. In this perspective, the method we envisage in this paper will be useful to trigger the electromagnetic follow-up of a GW detection, searching for the afterglow emission of a SGRBs.

Until then, the method we propose can be used in the data analysis of advanced detectors as follows:

- Table 2-3 indicate the systems that are more likely to produce accretion disks sufficiently massive to generate a SGRB. The table can be enriched including more NS equations of state or more binary parameters; however, it already contains a clear information on which is the range of parameters to be used in the GW data analysis, if the goal is to search for BH-NS signals which may be associated to a GRB. For instance, Table 2-3 suggests that searching for mass ratio smaller than, or equal to, 3 – 4, and values of the black-hole angular momentum larger than 0.5 – 0.6 would allow us to save time and computational resources in low-latency search. In addition, it would allow us to gain sensitivity in externally triggered searches performed in time coincidence with short GRBs observed by gamma-ray satellites.
- If a SGRB is observed sufficiently close to us in the electromagnetic waveband, the parameters of the GW signal detected in coincidence would allow us to set a threshold on the mass of the accretion disk. If the GW signal comes, say, from a

system with a BH with spin $\chi_{\text{BH}} = 0.5$, mass ratio $q = 7$, and neutron star mass $M_{\text{NS}} = 1.2M_{\odot}$, from Table 2, equations of state softer than the EoS 2H would be disfavored. Thus, we would be able to shed light on the dynamics of the binary system, on its parameters and on the internal structure of its components. We would enter into the realm of gravitational wave astronomy.

Finally, it is worth stressing that as soon as the fit (1) is extended to NS-NS coalescing binaries, this information will be easily implemented in our approach. With the rate of NS-NS coalescence higher than that of BH-NS, our approach will acquire more significance, and will be a very useful tool to study these systems.

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